Extra Class Math

Phil Sherrod – W4PHS

Direct versus Alternating Current

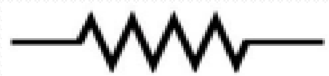
Things We Can Measure

Direct Current (DC)	Alternating Current (AC)
Voltage	Voltage (peak, RMS)
Current	Current (peak, effective)
Power	True power, Apparent power
Resistance	Resistance, Reactance, Impedance
---	Frequency
---	Phase

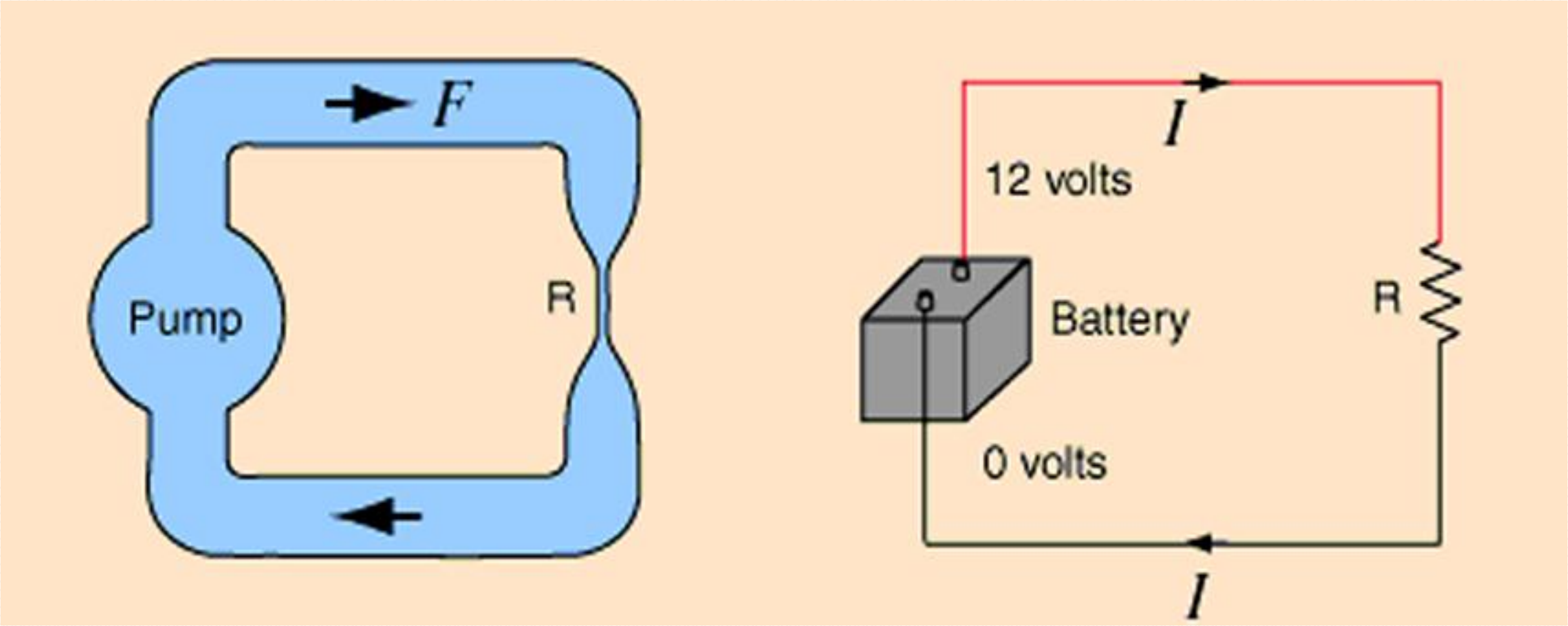
Resistance and Reactance

- **Resistance** (R) is simple – it "resists" or holds back the flow of current. Resistance comes from resistors.
- There are two types of **reactance** (X):
  - Capacitive reactance (X_C)
  - Inductive reactance (X_L)
- **Impedance** (Z) is the combined effect of resistance and reactance.
- **Admittance** (Y) is the reciprocal of impedance.

Resistors

- A resistor “resists” electric flow. It makes it hard for electricity to move through it.
- Electrical symbol: 
- Resistance is measured in Ohms.
- The greater the resistance (more Ohms), the harder it is for electricity to flow through it.
- A *pure* resistor does **not** change resistance value depending on frequency. However, most real resistors have some inductance and some capacitance.

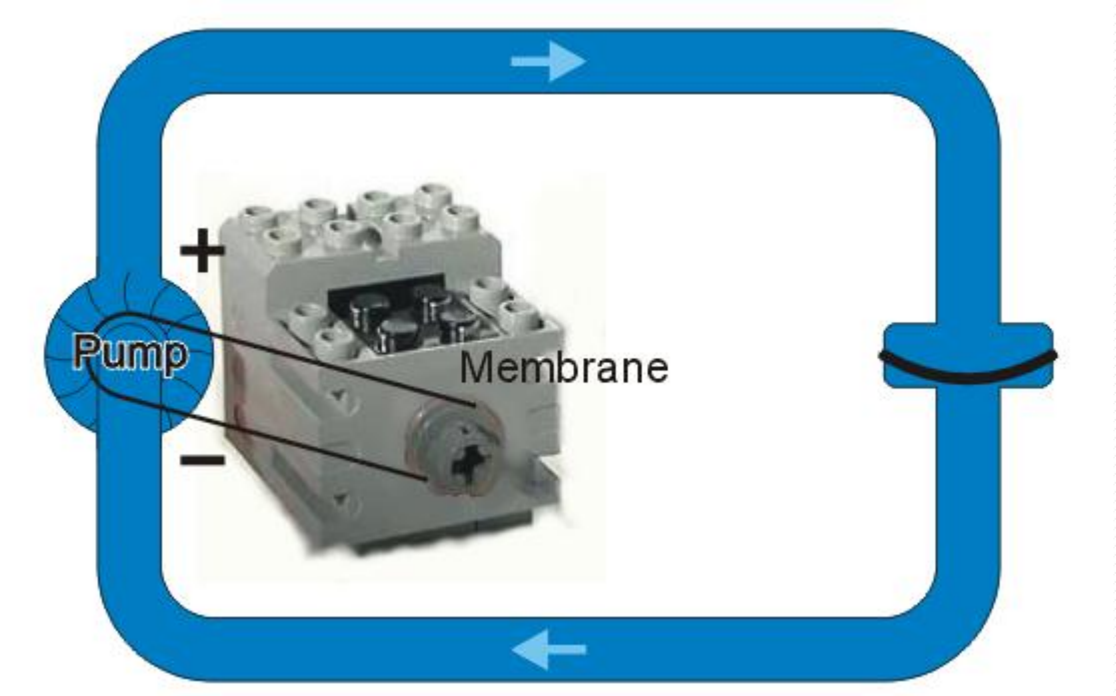
Water Analogy for Resistor



Capacitors

- A capacitor stores electricity like a rechargeable battery. They can be "recharged" trillions of times.
- Capacitance is measured in Farads.
- Capacitors **pass AC** current and **block DC**.
- Capacitive reactance (X_C) *decreases* with frequency.
- The lower the frequency, the greater the reactance.
- Electrical symbol: 

Water Analogy for Capacitor



Inductors

- An inductor is a coil of wire
- Inductors store energy in a magnetic field.
- Inductors **impede** the flow of **AC** but **pass DC**.
- Inductive reactance (X_L) increases with frequency
- As frequency increases, inductive reactance increases
- Inductance is measured in Henries
- All real inductors have some resistance
- Electrical symbol: 

Water Analogy for Inductor

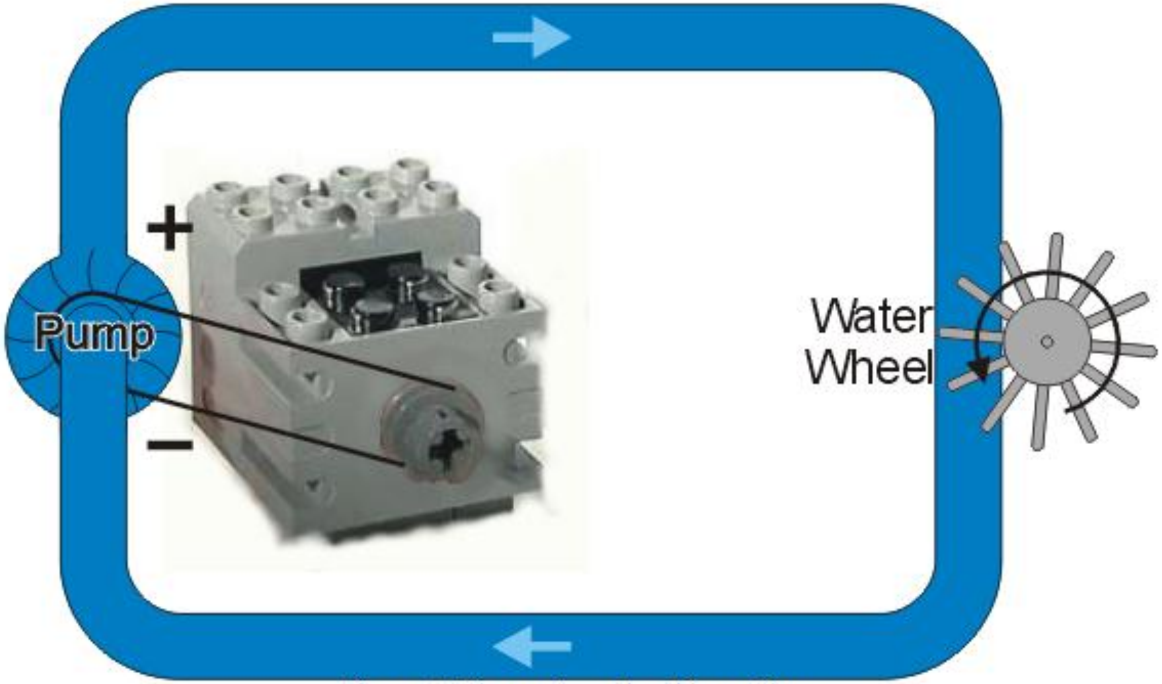


Figure 2. The water wheel in motion.

Powers of 10

- Giga = 10^9 (1,000,000,000)
- Mega = 10^6 (1,000,000)
- Kilo = 10^3 (1,000)
- Milli = 10^{-3} (0.001)
- Micro = 10^{-6} (0.000001)
- Nano = 10^{-9} (0.000000001)
- Pico = 10^{-12} (0.000000000001)

Entering Powers of 10 on Calculator

- Look for the "**EE**" key on the calculator.
- "EE" stands for Enter Exponent.
- Press "EE" then enter the power of 10.
- Press the +/- key after the value to negate it.
- Enter 2 megaohms: 2, EE, 6 = 2,000,000
- Enter 500 MHZ: 500, EE, 6 = 500,000,000
- Enter 23 millivolts: 23, EE, 3, +/- = 0.023
- Enter 19 picofarads: 19, EE, 12, +/- = 1.9^{-11}

Inductive Reactance (X_L)

- $X_L = 2 \cdot \pi \cdot f \cdot L$
- f = frequency in Hertz, L is inductance in Henrys.
- What is X_L for a 10 microhenry (10^{-6}) inductor at 500 MHz (10^{6})?
- $X_L = 2 \cdot \pi \cdot 500_{EE}6 \cdot 10_{EE} - 6$
- X_L = 31,416 Ω

Capacitive Reactance (X_C)

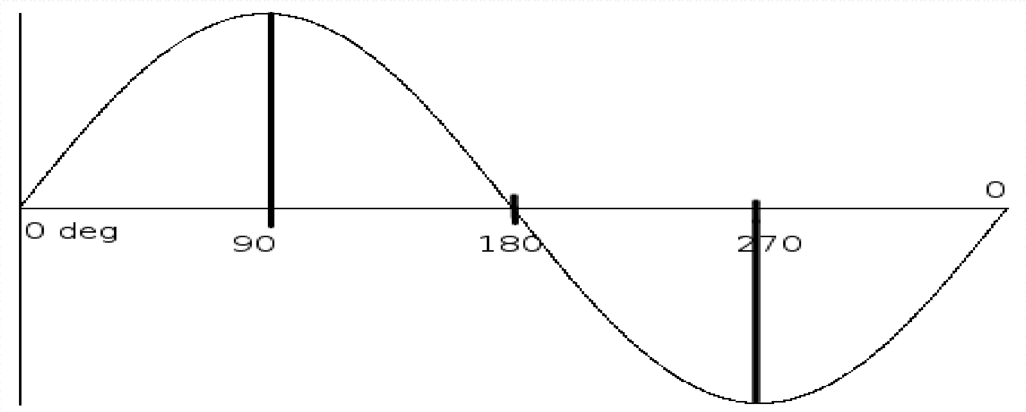
- $X_C = \dfrac{1}{2 \cdot \pi \cdot f \cdot C}$
- f = frequency in Hertz, C = Capacitance in Farads
- What is X_C for 19 pF (10^{-12}) capacitor at 21.2 MHz (10^6)?
- $X_C = \dfrac{1}{2 \cdot \pi \cdot 19_{EE}{}^{-12} \cdot 21.2_{EE}{}^{6}} = \dfrac{1}{0.002531} = 395\ \Omega$

Impedance: Resistance & Reactance

- **Impedance** (Z) is the combination of resistance (R) and reactance (X).
- Resistance and Reactance are kept separate and not added together.
- Impedance is written $\mathbf{Z = R \pm \mathit{j}\, X}$
- Where **R** is resistance and **X** is reactance
- Resistance is always positive (or zero)
- Reactance can be positive or negative or zero
- If R = 100 Ω, and X = 200 Ω, Z = 100 + *j*200 Ω

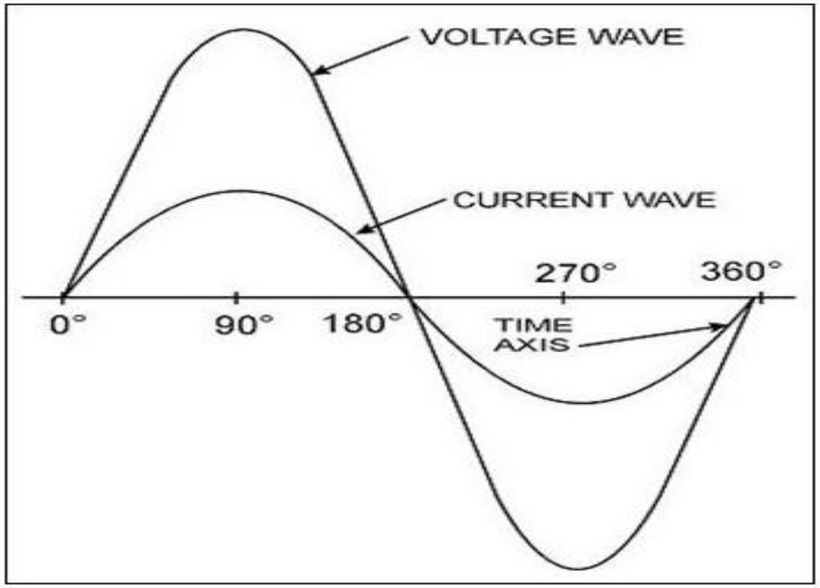
Electric Phase = Time Along Wave

- "**Phase**" is the position of a sine wave in time
- Phase is measured from 0 to 360 degrees
- The sine wave has values of 0 at 0, 180 and 360 degrees
- The peak positive value is at 90°
- The peak negative value is at 270 °



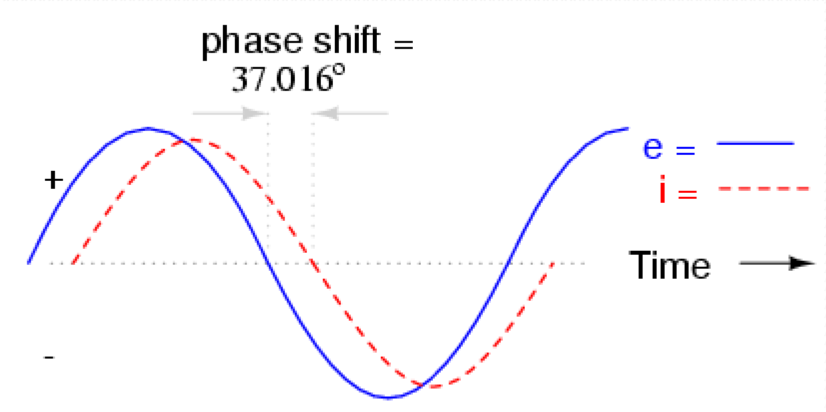
Current/Voltage Waves In-phase

If you put an AC current through a **resistor**, current and voltage are in phase. Amplitude is different, but peaks and zero crossings match.



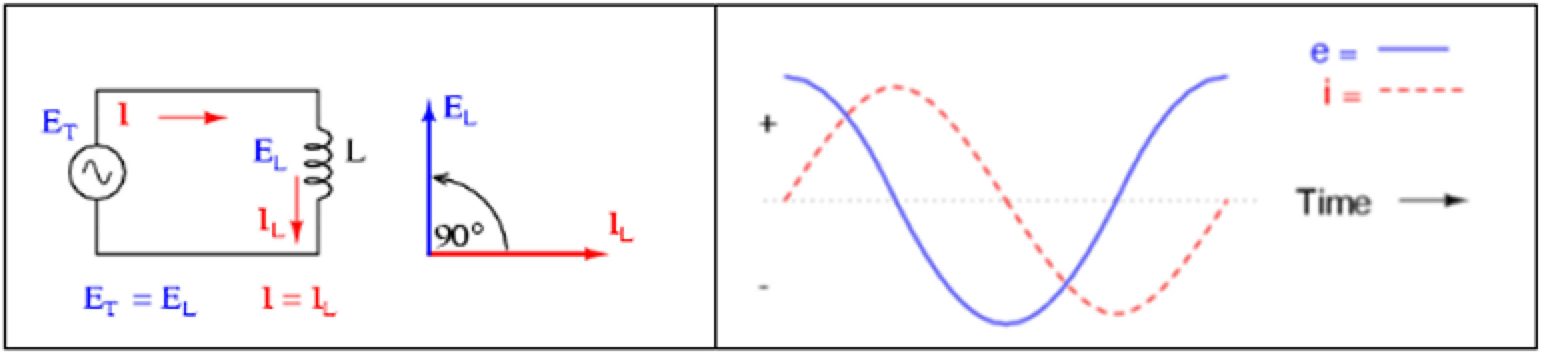
Phase Shifted Waves

- In AC circuits with **capacitors** and **inductors**, the phase can be shifted.
- The amount of shift is the "**phase shift**".
- Voltage can lead current or current can lead voltage.



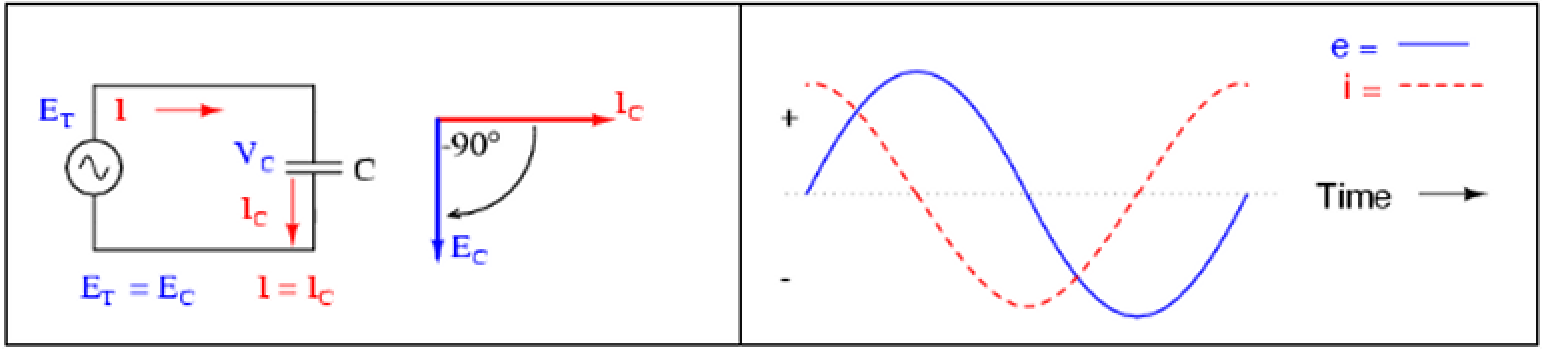
Inductor: Voltage leads Current

- If you measure voltage across an inductor and current through it, the voltage *leads* the current by 90°. The is the same as saying the current *lags* the voltage.



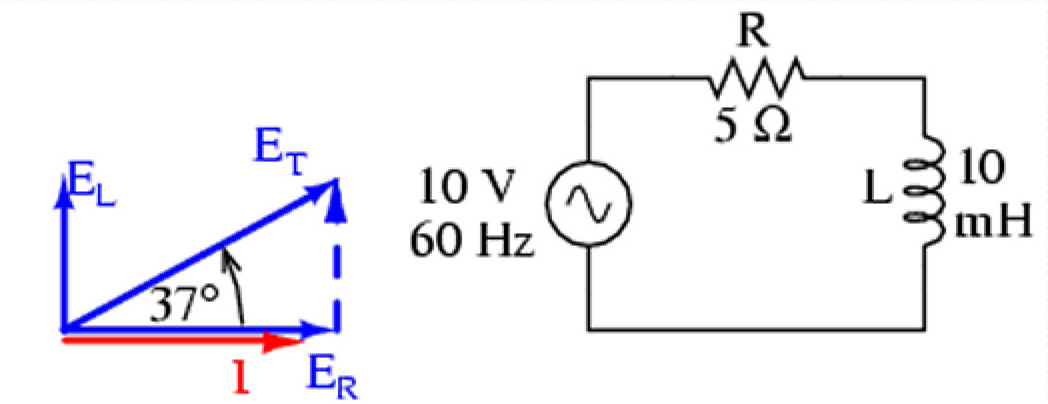
Capacitor: Current Leads Voltage

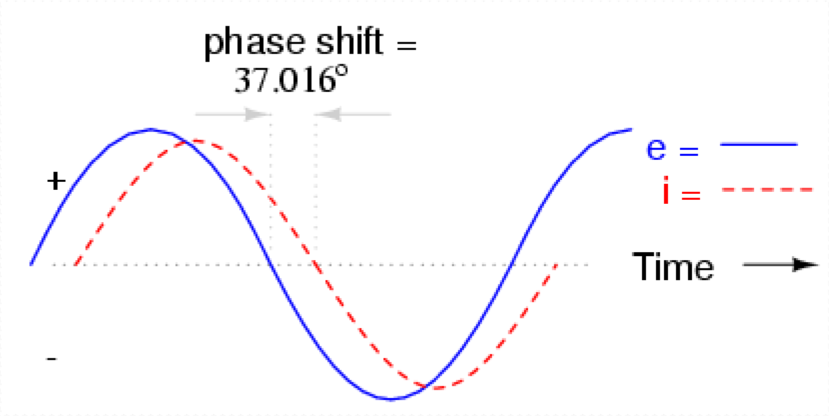
- If you put a voltage across a capacitor, the voltage *leads* the current by 90°. This is the same as saying the current *lags* the voltage by 90°.



Phase Shift May Not Be 90°

- If you have a resistor in the circuit with a capacitor or inductor, the phase shift will be less than 90°.





Here's How To Remember

"ELI the ICE man"

ELI	ICE
E = Voltage	I = Current
L = Inductor	C = Capacitor
I = Current	E = voltage
Voltage (E) *leads* Current (I)	Current (I) *leads* Voltage (E)
Current (I) *lags* Voltage (E)	Voltage (E) *lags* Current (I)
"ELI is hot", temperature is **+**	"ICE is cold", temperature is **-**
Inductive reactance X_L is $+j$	Capacitive reactance X_C is $-j$
Pure inductor, phase shift 90°	Pure capacitor, phase shift 90°

Exam Questions E5B09, E5B10

- E5B09: What is the relationship between the current through a capacitor and the voltage across a capacitor?
- Answer D: Current leads voltage by 90°. (“ICE”)

- E5B10: What is the relationship between the current through an inductor and the voltage across an inductor?
- Answer A: Voltage leads current by 90°. (“ELI”)

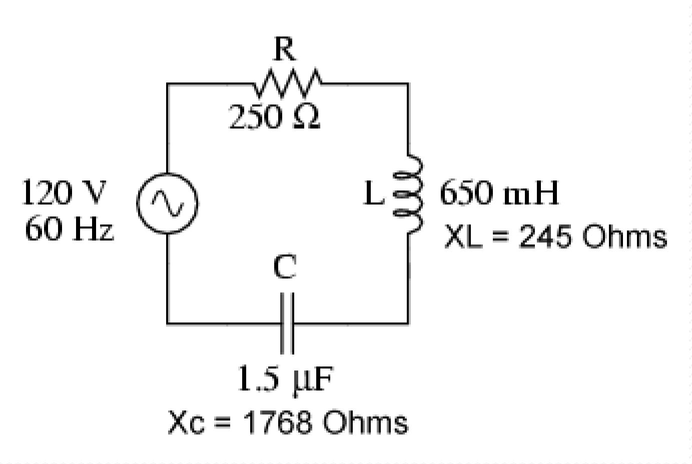
Practicing Impedance

- If $R = 100\ \Omega$ and $X_L = 200\ \Omega$, what is Z?
- Ans.: $Z = 100 + j200$ (Note, X_L is positive)

- If $R = 1000\ \Omega$ and $X_C = 500\ \Omega$, what is Z?
- Ans.: $Z = 1000 - j500$ (Note, X_C is negative)

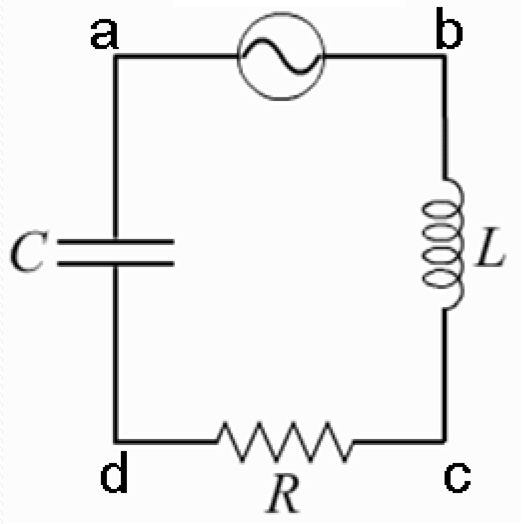
Series Impedance

- If two or more components are in *series*, the total impedance (Z) is the sum of the resistance and reactance.
- X_L is +, X_C is –
- R = 250
- $X_L = j245$
- $X_C = -j1768$
- $Z = 250 - j1523$



Resonance

- In a circuit with an inductor and a capacitor, resonance occurs when X_L equals X_C, because they cancel each other out.
- Remember, X_L is positive and X_C is negative
- Example, R=50, $X_L = +j100$, $X_C = -j100$, Z = 50

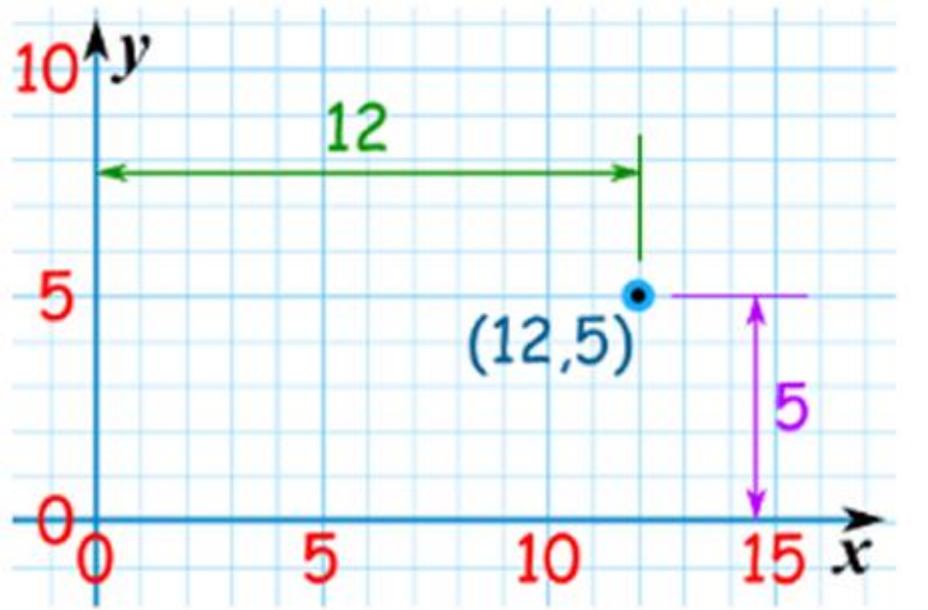


Calculating Resonant Frequency

- Resonance occurs when $X_L = X_C$
- $X_L = 2 \cdot \pi \cdot f \cdot L$
- $X_C = 1/(2 \cdot \pi \cdot f \cdot C)$
- Resonance when $X_L = X_C$
- Resonance when $2 \cdot \pi \cdot f \cdot L = 1/(2 \cdot \pi \cdot f \cdot C)$
- $\boldsymbol{FR = \dfrac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}}$

Rectangular Coordinates

- Position is specified as a pair of numbers (*x*,*y*)
- X is the horizontal axis. Y is the vertical axis.
- X and Y coordinates can be negative.
- Here is a point at (12,5) [x=12, y=5]

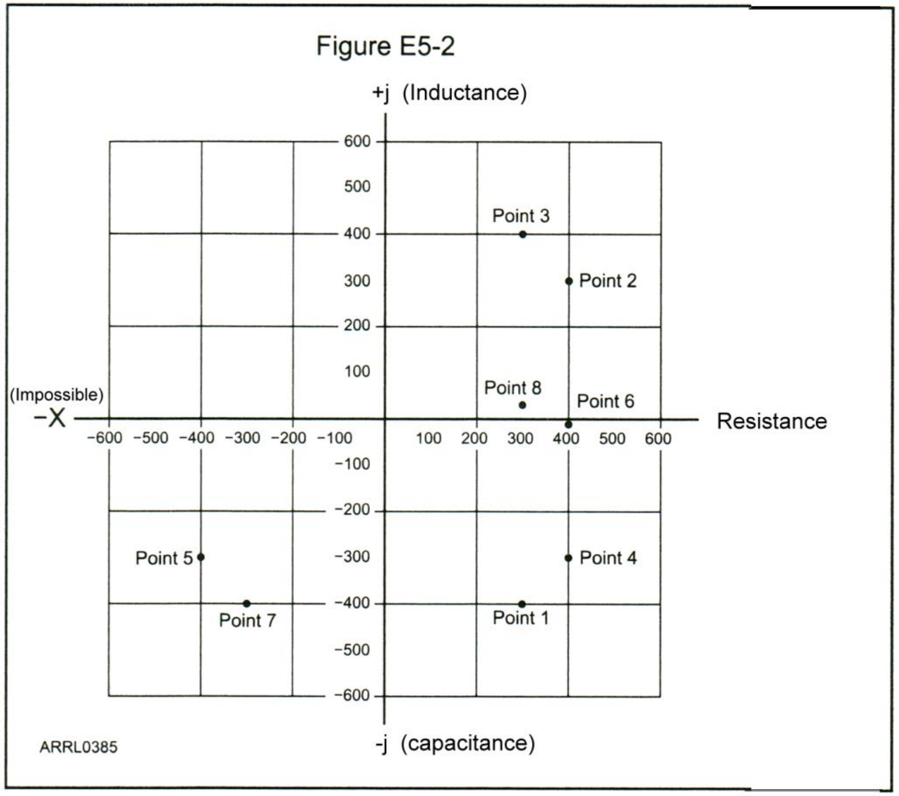


Plotting Impedance Values

- Impedance (Z) has two components:
  - Resistance (R) which is always positive or zero
  - Reactance (X) which can be positive or negative
    Inductance is ***+j***   Capacitance is ***–j***
    Remember, **ELI the ICE man** (ELI is hot, ICE is cold)
  - Written $Z = R \pm jX$
  - Examples: 100 +*j*200   50 –*j*75
- Impedance can be plotted using rectangular coordinates for resistance and reactance.
- Later we will learn how to write impedance using polar coordinates – magnitude and angle.

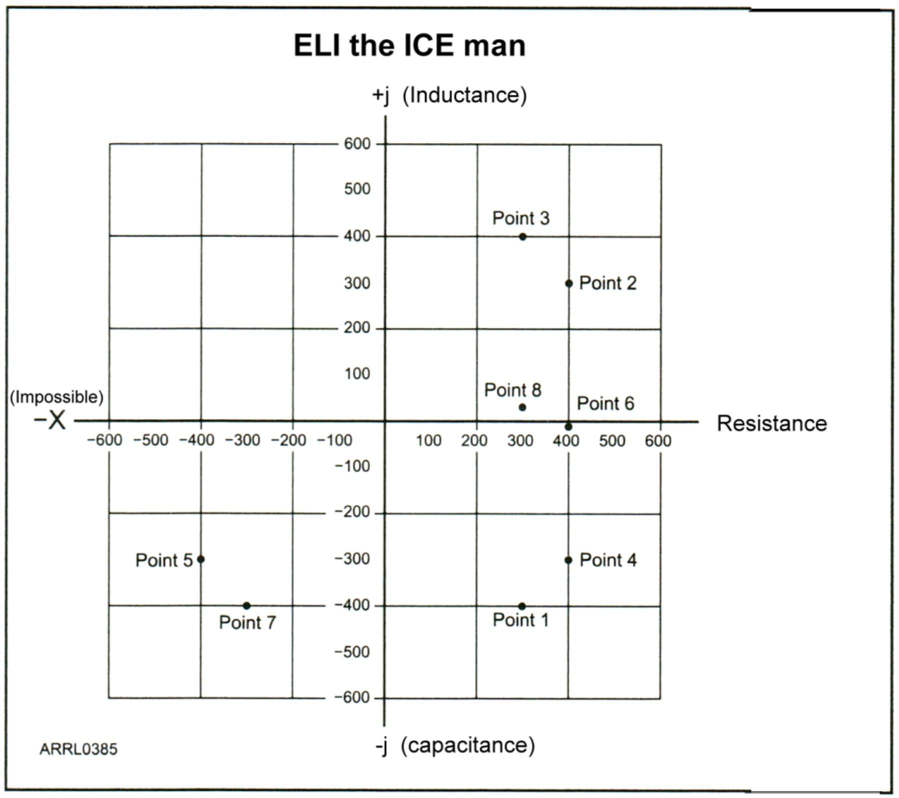
Resistance, Reactance Plane

- The vertical ***j*** axis is reactance
  (+ inductive, - capacitive) – E**L**I the I**C**E man
- The horizontal axis is resistance (can't be negative)
- Figure E5-2 pg. 13-55



Impedance Chart Exercise

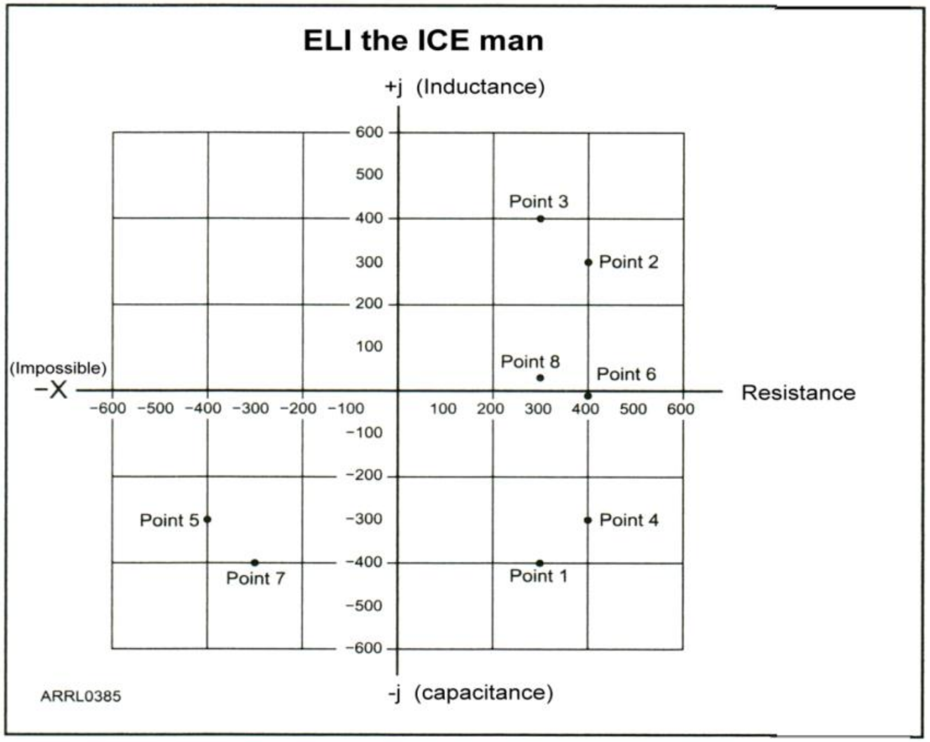
- Write down impedance for each point $Z = R \pm jX$
- Which points are impossible?
- Which points are inductive and which are capacitive?
- Figure E5-2 pg. 13-55



Exam Question E5C19 (pg. 13-55)

- E5C19: Which point on Figure E5-2 best represents that impedance of a series circuit consisting of a 400 ohm resistor and a 38 picofarad (38_{EE}-12) capacitor at 15 MHz ($15_{EE}6$)?

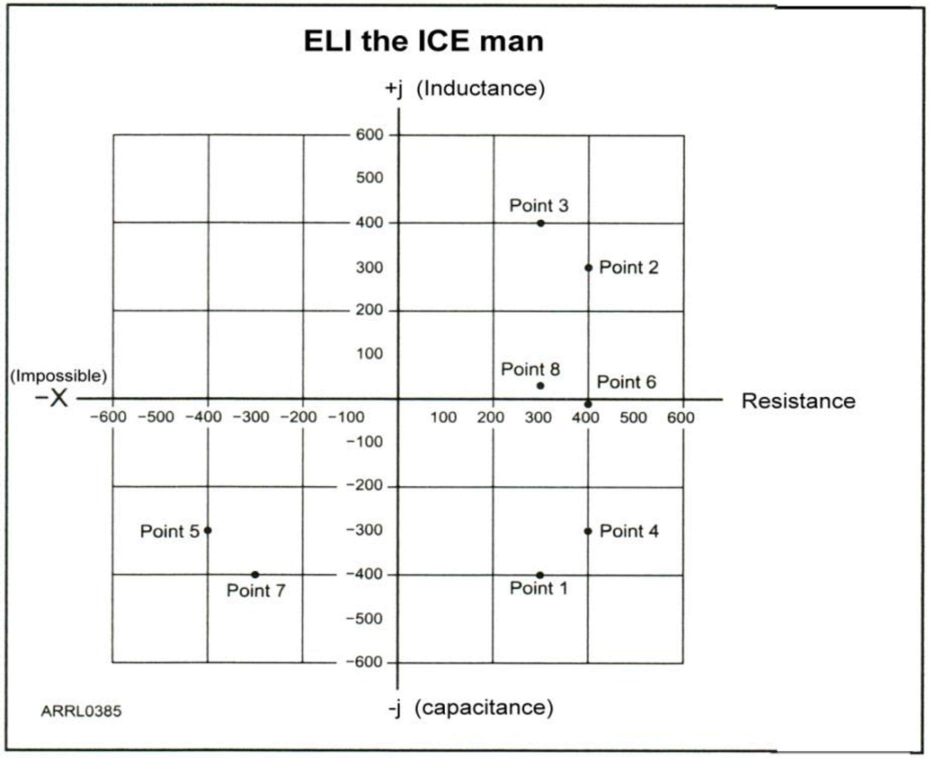
- $X_C = \frac{1}{2 \cdot \pi \cdot f \cdot C}$
- $Xc = 1/(2 \cdot \pi \cdot 38_{EE}\text{-}12 \cdot 15_{EE}6)$
- $X_C = 279.2\ \Omega$
- $Z = 400 - j279$
- Answer: Point 4



Exam Question E5C20 (pg. 13-55)

- E5C20: Which point in Figure E5-2 best represents the impedance of a series circuit consisting of a 300 ohm resistor and an 18 microhenry (18_{EE}-6) inductor at 3.505 MHz ($3.505_{EE}6$)?

- $X_L = 2 \cdot \pi \cdot f \cdot L$
- $X_L = 2 \cdot \pi \cdot 3.505_{EE}6 \cdot 18_{EE}\text{-}6$
- $X_L = 395.4\ \Omega$
- $Z = 300 + j395$
- Answer: Point 3

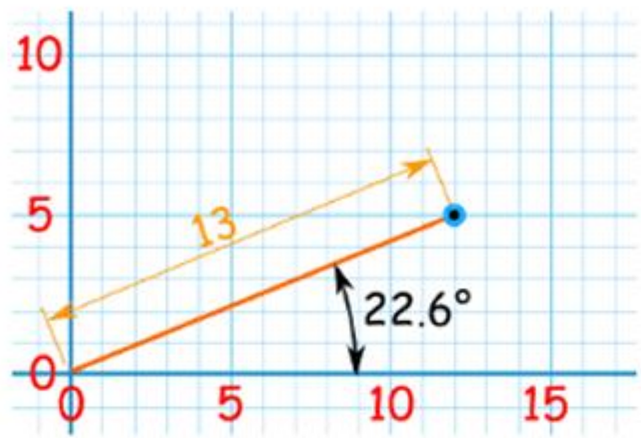


Exam Question E5C22 (pg. 13-56)

- E5C22: In rectangular coordinates, what is the impedance of a network consisting of a 10 microhenry inductor in series with a 40 ohm resistor at 500 MHz?
- X_L = 10 microhenry inductor at 500 MHz
- $X_L = 2 \cdot \pi \cdot f \cdot L$
- $X_L = 2 \cdot \pi \cdot 500_{EE}6 \cdot 10_{EE}{-6} = 31416$
- Z = 40 + j31416
- Ans. A 40 + *j*31,400

Polar Coordinates

- A point is specified by a distance from the origin and a rotation angle. Written: *magnitude ∠ angle*
- An angle of 0° is on the +X axis.
- Rotation is in a counterclockwise direction.
- Impedance can be in rectangular or polar coordinates



The polar coordinates of the point are 13 ∠ 22.6°.

Quick Tips – Solve Without Math!

- Rectangular: $Z = R \pm jX$ Polar: $Z = |Z| \angle \pm\theta$
- $|Z|$ is *magnitude* of impedance, θ is angle
- $|Z|$ is always greater than R or X
- $|Z|$ is always less than $R + X$
- If $R = X$ then $\theta = \pm 45°$ $+\theta$ if $+X$, $-\theta$ if $-X$
- If R *(resistance)* > X *(reactance)* then $\theta < 45°$
- If X *(reactance)* > R *(resistance)* then $\theta > 45°$
- If reactance is inductive (+), angle is positive
- If reactance is capacitive (-), angle is negative
- If resistance and reactance are 300 and 400, $|Z| = 500$
- If 14° is a possible answer choice, it is always right.

Exam Question E5C01 (pg. 13-52)

- E5C01: In polar coordinates, what is the impedance of a network consisting of a 100 ohm reactance inductor in series with a 100 ohm resistor.
- $Z = 100 + j100$
- Since resistance = reactance, the angle must be 45°
- $|Z|$ is > 100 and < 200
- Answer is B: 141 ohms at an angle of 45 degrees

Exam Question E5C02 (pg. 13-52)

- E5C02: in polar coordinates, what is the impedance of a network consisting of a 100 ohm reactance inductor, a 100 ohm reactance capacitor, and a 100 ohm resistor all connected in series?
- $X_L = +j100$
- $X_C = -j100$
- $X_{TOTAL} = +j100 - j100 = 0$
- Z = 100 Ω pure resistance
- Point is on the horizontal (resistance) axis
- Answer D: 100 ohms at an angle of 0 degrees

Exam Question EC503 (pg. 13-52)

- EC503: In polar coordinates, what is the impedance of a network consisting of a 300 ohm reactance capacitor, a 600 ohm reactance inductor, and a 400 ohm resistor, all connected in series?
- $X_L = +j600$
- $X_C = -j300$
- $X_{TOTAL} = +j600 - j300 = +j300$ (inductive, positive $\angle$)
- $Z = 400 + j300$
- Since $R > X$, angle $< 45°$
- $|Z|$ (magnitude) > 400 and < 700 $(3, 4 = 5)$
- Answer A: 500 ohms at 37 degrees

Exam Question E5C04 (pg. 13-52)

- E5C04: In polar coordinates, what is the impedance of a network consisting of a 400 ohm reactance capacitor in series with a 300 ohm resistor?
- R = 300
- $X_C = -j400$ (capacitor = negative, so angle is negative)
- $Z = 300 - j400$
- $|Z|$ (magnitude) > 400 and < 700 (3, 4 = 5)
- Angle is > -45° (because reactance > resistance)
- Answer D: 500 ohms at an angle of -53.1 degrees

Exam Questions E5C06, E5C08

- E5C06: In polar coordinates, what is the impedance of a network consisting of a 100 ohm reactance capacitor in series with a 100 ohm resistor?
- Answer D: 141 ohms at -45 degrees

- E5C08: In polar coordinates, what is the impedance of a network comprised of a 300 ohm reactance inductor in series with a 400 ohm resistor?
- Answer B: 500 ohms at an angle of 37 degrees

Exam Question E5B11, E5B12

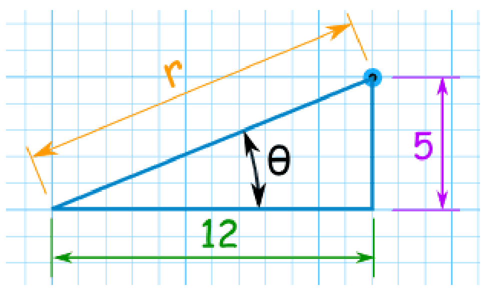
- E5B11: What is the phase angle between the voltage across and the current through a series RLC circuit if X_C is 25 ohms, R is 100 ohms, and X_L is 50 ohms?
- $Z = 100 + j25$ (R > X so angle < 45°)
- Answer B: 14° with voltage leading current (ELI)

- E5B12: What is the phase angle between the voltage across and the current through a series RLC circuit if X_C is 75 ohms, R is 100 ohms, and X_L is 50 ohms?
- $Z = 100 - j25$ (R > X so angle < 45°)
- Answer C: 14° with voltage lagging current (ICE)

Exam Question E5C18 (pg. 3-54)

- E5C18: In polar coordinates, what is the impedance of a series circuit consisting of a resistance of 4 ohms, an inductive reactance of 4 ohms, and a capacitive reactance of 1 ohm?
- $X_L = +j4$
- $X_C = -j1$
- Reactance adds: $X = +j4 - j1 = +j3$
- $Z = 4 + j3$ (positive reactance, so inductive)
- $|Z| = 5$ because 3, 4 → 5 $|Z| = \sqrt{3^2 + 4^2} = 5$
- Since $R > X$, angle $< 45°$
- Answer: **B** 5 ohms at 37 degrees

Rectangular to Polar Conversion

- 

	$r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}(y/x)$

- Rectangular: $x = 12$ $y = 5$ Polar: $|Z| \angle \theta$
- Resistance (R) on X axis = 12
- Inductance (X) on Y axis = 5
- $|Z| = \sqrt{5^2 + 12^2} = 13$ ($|Z|$ is hypotenuse: r)
- $\theta = \tan^{-1}(5/12) = 22.62°$
- $(5, 12) \rightarrow 13 \angle 22.62°$

Exam Question E5C15 (pg. 13-54)

- E5C15: In polar coordinates, what is the impedance of a circuit of 100 –*j*100 ohms impedance?
- Resistance (R) = 100
- Reactance (X) = -*j*100
- Z = 100 –*j*100
- $|Z| = \sqrt{100^2 + (-100)^2}$ = 141.2 Ω
- θ = $\tan^{-1}(-100/100)$ = -45°
- Z = 141.2 ∠ -45°
- Answer: **A** 141 ohms at an angle of -45 degrees

Exam Question E5C18

- E5C18: In polar coordinates, what is the impedance of a series circuit consisting of a resistance of 4 ohms, an inductive reactance of 4 ohms, and a capacitive reactance of 1 ohm?
- $Z = 4 + j4 - j1 = 4 + j3$
- $|Z| = \sqrt{4^2 + 3^2} = 5\ \Omega$
- $\theta = \tan^{-1}(3/4) = 36.87°$
- $Z = 5 \angle 37°$
- Answer: **B** 5 ohms at an angle of 37 degrees

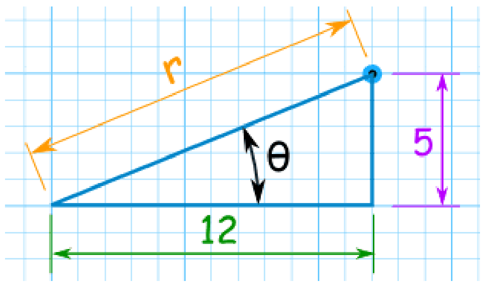
Admittance in Polar Coordinates

- "Admittance" is the opposite of impedance. The larger the value, the easier it is for current to flow.
- Polar form: $Y = |Y| \angle \pm\theta$
- Units = "Siemens"
- How to convert polar admittance to polar impedance:
  - Compute inverse of magnitude $|Y|$
  - Negate the angle, θ
- $Z = 1/|Y| \angle -\theta$

Exam Question E5C16 (pg. 13-54)

- E5C16: In polar coordinates, what is the impedance of a circuit that has an admittance of 7.09 millisiemens at 45 degrees?
- $Y = 7.09_{EE}\text{-}3 \angle 45$
- $Z = 1/7.09_{EE}-3 \angle -45$
- $Z = 141 \angle -45$
- Ans. **B**: 141 ohms at an angle of -45 degrees

Polar to Rectangular Conversion

- 

$R = |Z| \cos(\theta)$

$X = |Z| \sin(\theta)$

- Polar: $13 \angle 22.62°$
- $|Z| = 13 \quad \theta = 22.62°$
- $\mathbf{R} = 13 \cdot \cos(22.6) = 12$
- $\mathbf{X} = 13 \cdot \sin(22.6) = 5$ (since θ is +, X is +)
- $13 \angle 22.62° \rightarrow (12, 5)$

Exam Question E5C17 (pg. 13-54)

- E5C17: In rectangular coordinates, what is the impedance of a circuit that has an *admittance* of 5 millisiemens at -30 degrees?
- Polar Admittance (Y) = $5_{EE}{-3} \angle -30°$
- Reciprocal of magnitude = $1/5_{EE}{-3}$ = 200 Ω
- Polar: 200 ∠ 30 (note, we reversed sign of angle)
- R = 200 · cos(30) = 173.2 Ω
- X = 200 · sin(30) = 100 Ω (X is + because θ is +)
- Z = 173 + *j*100
- Ans. **C**: 173 + *j*100

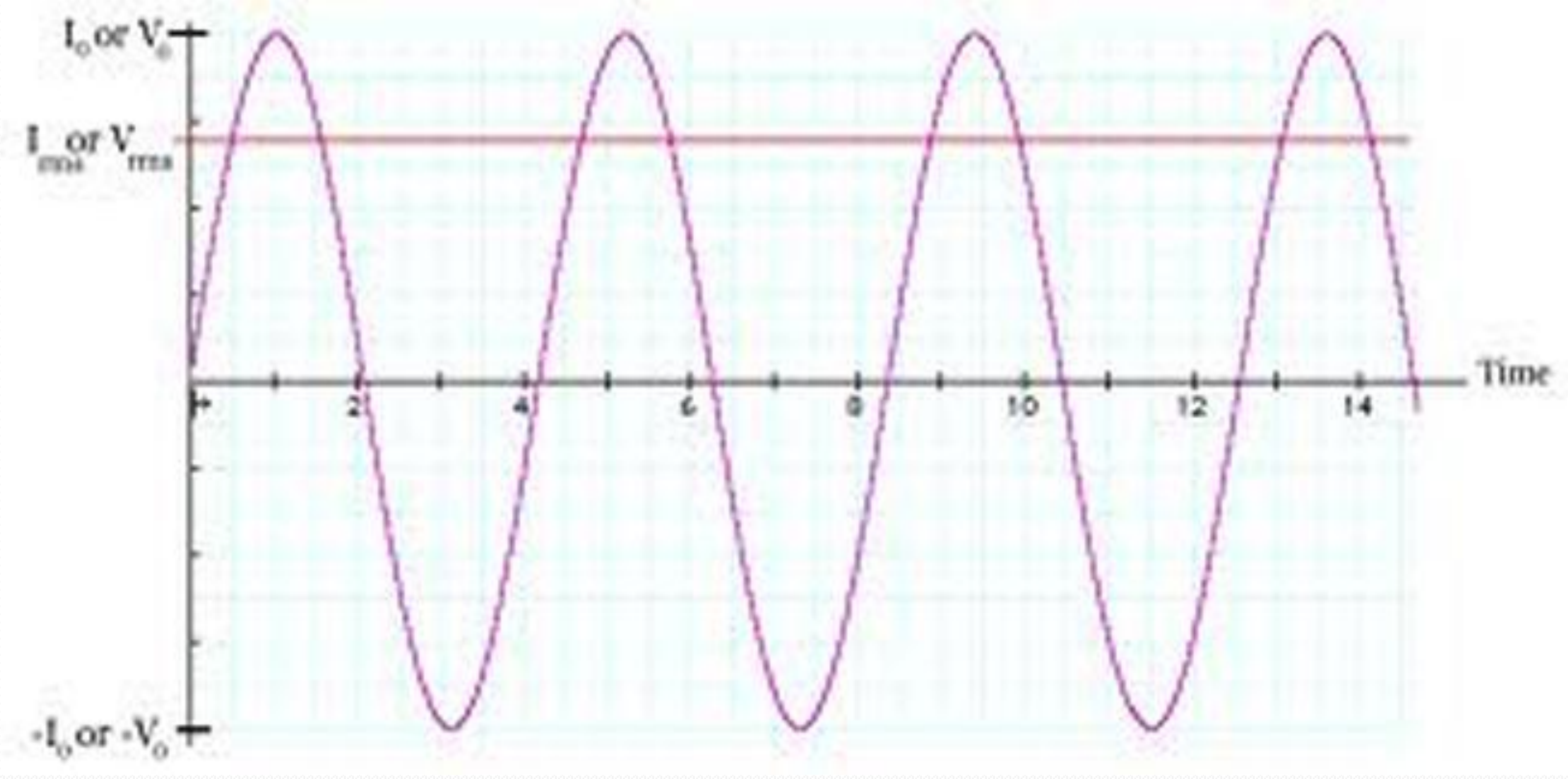
Power Factor

- In DC circuits, power calculation is easy:
- Power (watts) = E (volts) · I (amps)
- If there is a 12 volt drop (E) across a resistor carrying 10 amps (I), then Power = 12 · 10 = 120 watts
- Power calculation in AC circuits is more complex:
  - Voltage (E) is changing with time
  - Current (I) is changing with time
  - Voltage and current may be out of phase

Power in AC Circuits

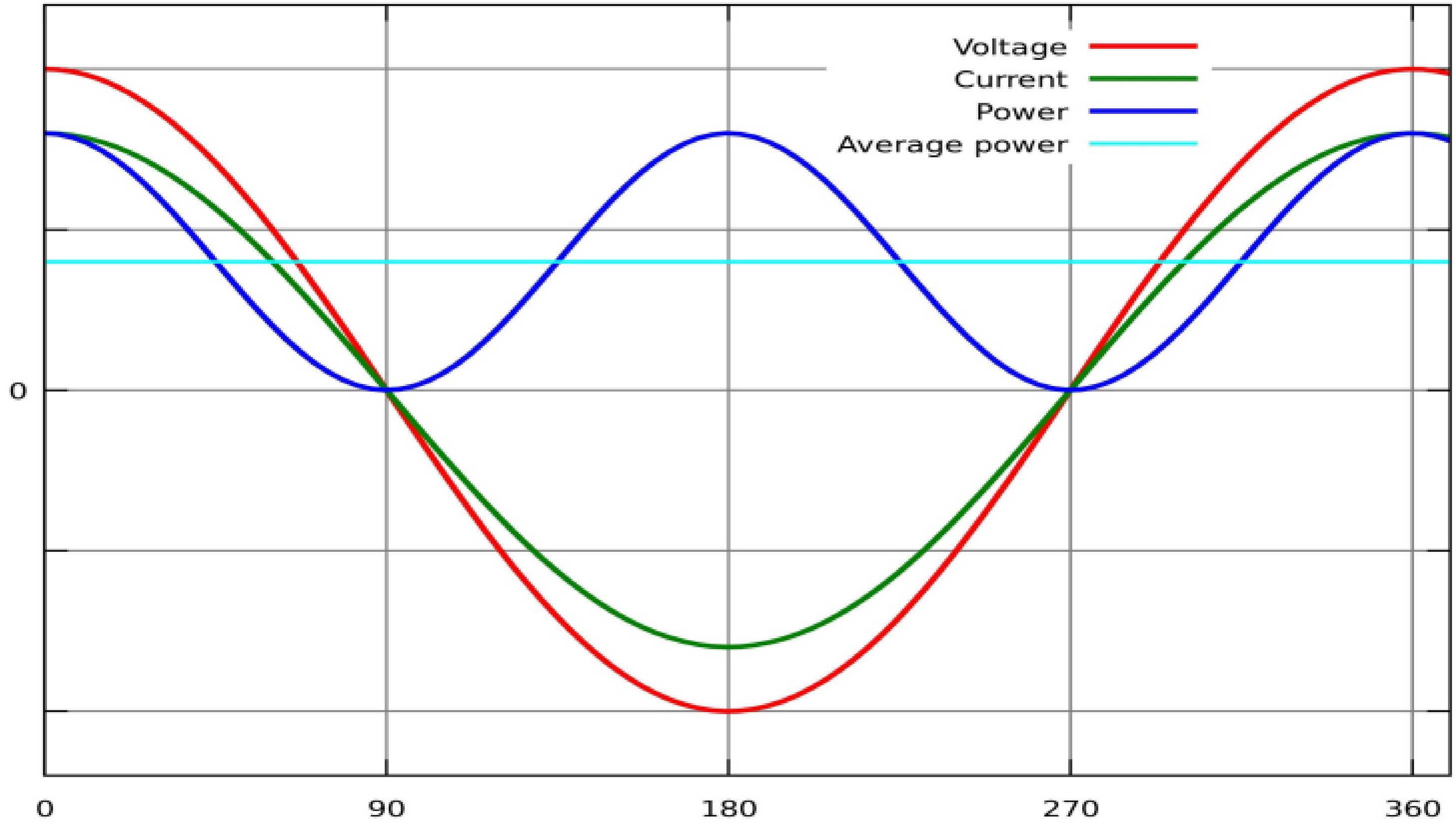
- Root Mean Square (RMS) voltage:
- RMS voltage is the equivalent AC voltage that causes the same heating in a resistor as a constant DC voltage.
- For a sine wave:
- $E_{RMS} = 0.707 \cdot E_{peak} = 0.353 \cdot E_{peak\ to\ peak}$
- $E_{peak} = 1.41 \cdot E_{RMS}$ $E_{peak\ to\ peak} = 2 \cdot 1.41 \cdot E_{RMS}$
- If E_{peak} is 170 volts, $E_{RMS} = 0.707 \cdot 170 = 120$ volts
- If E_{RMS} is 120, $E_{peak\ to\ peak}$ is $2 \cdot 1.41 \cdot 120 = 339$ volts

RMS Voltage for Sine Wave



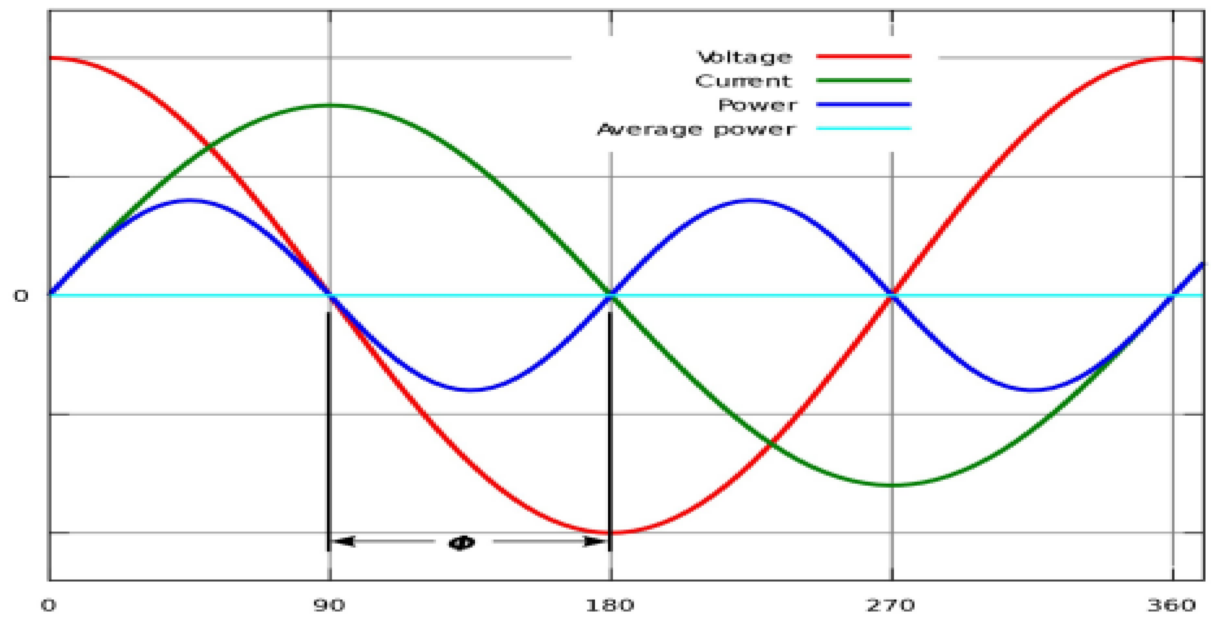
Power Factor with E and I In Phase

- "Simple" case: voltage and current are in phase
- Instantaneous Power = E · I (just like DC)



Power Factor, E and I Out of Phase

- Instantaneous Power = E · I · Power factor
- Power factor depends on amount of phase shift

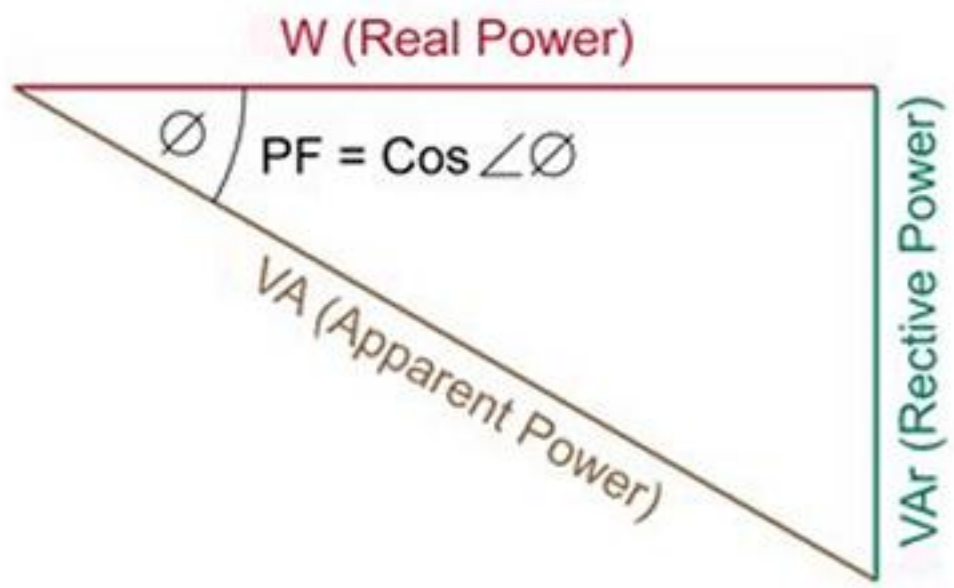


Different Types of "Power"

- **Real Power (W)** = Power that actually does work.
- **Apparent Power (VA)** is simply $E \cdot I$. This is also called "Total Power".
- **Reactive Power (VAR)** is power through inductor or capacitor. It is stored in the inductor or capacitor but does not do any actual work in the circuit load.
- Real power is measured in watts
- Apparent power is measured in volt-amps
- Reactive power is measured in volt-amps-reactive
- Only resistors use real power

Calculating Power Factor

$$\text{Power Factor (pf)} = \frac{\text{W (Real Power)}}{\text{VA (Total Power)}}$$



Calculating Power Factor

- Power Factor (PF) = $\frac{\text{Real Power}}{\text{Apparant Power}}$
- Real power = PF · Apparant power
- Power factor = cos(θ)
- Note: cos(90°) = 0.0
- No real power dissipated if phase shift = 90°
- Note: cos(0°) = 1.0 (PF = 1.0 if no phase shift)
- If voltage and current are in phase, power factor = 1.0 and real power = apparent power (pure resistance)

Exam Question E5D12 (pg. 13-58)

- E5D12: How many watts are consumed in a circuit having a power factor of 0.2 if the input is 100 volts at 4 amps?
- Apparent power (VA) = $100 \cdot 4 = 400$ VA
- $W = PF \cdot VA$
- $W = 0.2 \cdot 400 = 80$ watts
- Ans. **B**: 80 watts

Exam Question E5D11 (pg. 13-57)

- E5D11: What is the power factor of an R-L circuit having a 60 degree phase angle between the voltage and the current:
- PF = cos(60) = 0.5
- Ans. **C**: 0.5